A.P. State Council of Higher Education
Semester-wise Revised Syllabus under CBCS, 2020-21

Course Code:

Four-year B.A. /B.Sc. (Hons)
Domain Subject: **MATHEMATICS**
IV Year B.A./B.Sc.(Hons)– Semester – V

Max Marks: 100

Course-6B: Multiple Integrals and Applications of Vector Calculus
(Skill Enhancement Course (Elective), 5 credits)

I. Learning Outcomes:

Students after successful completion of the course will be able to

1. Learn multiple integrals as a natural extension of definite integral to a function of two variables in the case of double integral / three variables in the case of triple integral.
2. Learn applications in terms of finding surface area by double integral and volume by triple integral.
3. Determine the gradient, divergence and curl of a vector and vector identities.
4. Evaluate line, surface and volume integrals.
5. understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral (Green's theorem), relation between line and surface integral (Stokes theorem)

II. Syllabus: (Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Multiple Integrals (15h)

1. Introduction, Double integrals, Evaluation of double integrals, Properties of double integrals.
2. Region of integration, double integration in Polar Co-ordinates, change of order of integration.
3. Triple integral, region of integration, Evaluation of triple integrals

Unit – 2: Vector Differentiation- I (15h)

1. Vector differentiation, ordinary derivatives of vectors, partial differentiation.
2. Gradient of a scalar point function, Directional derivative, Angle between two surfaces.

Unit – 3: Vector Differentiation - II (15h)

1. Divergence, Curl and Laplacian operator.
2. Formulae involving these operators.

Unit – 4: Vector Integration (15h)

1. Line Integrals with examples.
2. Surface Integral with examples.
3. Volume integral with examples.

Unit – 5: Vector Integration Applications (15h)

1. Gauss theorem and applications of Gauss theorem.
2. Green's theorem in plane and applications of Green's theorem.
3. Stokes's theorem and applications of Stokes theorem.

III. Text Book: Multiple Integrals and Application of Vector Calculus by Dr. A. Anjaneyulu, Deepthi Publications, Tenali.

Reference Books:

1.Dr.M Anitha, Linear Algebra and Vector Calculus for Engineer, Spectrum University Press, SR Nagar, Hyderabad-500038, INDIA.

2.Dr.M.Babu Prasad, Dr.K.Krishna Rao, D.Srinivasulu, Y.AdiNarayana, Engineering Mathematics-II, Spectrum University Press, SR Nagar, Hyderabad-500038,INDIA.

3. V.Venkateswararao, N. Krishnamurthy, B.V.S.S.Sarma and S.Anjaneya Sastry, A text Book of B.Sc., Mathematics Volume-III, S. Chand & Company, Pvt. Ltd., Ram Nagar,NewDelhi-110055.

4. R.Gupta,Vector Calculus, Laxmi Publications.

5. P.C.Matthews, Vector Calculus, Springer Verlag publications.

6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory:

1. **For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).

   1. The methods of evaluating double integrals and triple integrals in the class room and train to Evaluate These integrals of different functions over different regions.

   2. Applications of line integral, surface integral and volume integral.

   3. Applications of Gauss divergence theorem, Green's theorem and Stokes's theorem.

2. **For Student: Fieldwork/Project work** Each student individually shall undertake Fieldwork/Project work and submit areport not exceeding 10 pages in the given format on the work-done in the areas like thefollowing**,** by choosing any one of the following aspects.

   1. Going through the web sources like Open Educational Resources to find the values of double and triple integrals of specific functions in a given region and make conclusions. (or)

   2. Going through the web sources like Open Educational Resources to evaluate line integral, surface integral and volume integral and apply Gauss divergence theorem, Green's theorem and Stokes theorem and make conclusions.

3. **Suggested Format for Fieldwork/Project work Report**: Title page, Student Details, Index page,Stepwise work-done, Findings, Conclusions and Acknowledgements.

4. **Unit tests (IE).**

b) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area

V. Suggested Question Paper Pattern:

Max.Marks:75 **Time:3 hrs**

SECTION – A

(Answer any **five questions**. Each answer carries 5 Marks) 5 X 5=25

1) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dxdy$

2) Evaluate $\int_{x=0}^{a} \int_{y=0}^{x} \int_{z=0}^{x+y} e^{x+y+z}\,dxdydz$

3) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ where $x = \frac{2t+1}{t-1}, y = \frac{t^2}{t-1}, z = t+2$ then prove that $\left[\bar{r}' \quad \bar{r}'' \quad \bar{r}'''\right] = 0$.

4) Find the directional derivative of $\Phi = x^2-2y^2+4z^2$ at (1, 1,-1) in the direction of the vector $2\bar{i} + \bar{j} - \bar{k}$.

5) If $\bar{f} = \text{grad}\,(x^3+y^3+z^3-3xyz)$ then find div $\bar{f}$ and curl $\bar{f}$.

6) Find $\nabla^2\left(\frac{1}{r}\right)$

7) Evaluate $\int_C \bar{F} \bullet d\bar{r}$ where $\bar{F} = xy\,\bar{i} + yz\,\bar{j} + zx\,\bar{k}$ and the curve $\bar{r} = t\,\bar{i} + t^2\,\bar{j} + t^3\bar{k}$, t varying from -1 to 1.

8) Evaluate $\iiint (2x+y)\,dV$ where V is the closed region bounded by x=0, y=0, z=0, and x=2, y=2, z=4-x^2 .

9) If $\bar{F} = 2xy\,\bar{i} + yz\,\bar{j} + x^2\,\bar{k}$, find $\int_s \bar{F}.\bar{N}\,ds$ where S is the surface of the cube bounded the planes x=0, x=a, y=0, y=a, z=0, z=a by the application of Gauss divergence theorem.

10) Find $\oint_C (3x^2-8y^2)dx + (4y-6xy)dy$ by Green's theorem where C is the boundary defined by x=0, y=0, x+y=1.

SECTION - B

(Answer ALL the questions. Each question carries 10 Marks) 5 X 10 =50

11) Evaluate $\int_{y=0}^{a}\int_{x=0}^{\sqrt{a^2-y^2}}\left(x^2+y^2\right)dydx$ by changing the order of integratin.

(OR)

12) Evaluate $\iiint_V (x^2+y^2+z^2)\,dxdydz$ where V is the volume of the cube bounded by the coordinate planes and the planes $x=a, y=a, z=a$.

13) If $\bar{r}=2t\,\bar{i}+t^2\bar{j}+\frac{t^3}{3}\bar{k}$ then prove that $\frac{\left[\bar{r}'\ \bar{r}''\ \bar{r}'''\right]}{\left(\bar{r}'\times\bar{r}''\right)^2}=\frac{\left|\bar{r}'\times\bar{r}''\right|}{\left|\bar{r}'\right|^3}$ at $t=1$.

(OR)

14) If $a=x+y+z$, $b=x^2+y^2+z^2$, $c=xy+yz+zx$ then prove that $[\nabla a\ \nabla b\ \nabla c]=0$.

15) Prove that i) $\text{div}(\phi\,\bar{A})=(\text{grad}\,\phi).\bar{A}+\phi\,\text{div}\bar{A}$ ii) $\text{div}(\bar{A}\times\bar{B})=\bar{B}.\text{curl}\bar{A}-\bar{A}.\text{curl}\bar{B}$.

(OR)

16) Prove that div $\left\{\left(\bar{r}\times\bar{a}\right)\times\bar{b}\right\}=-2\left(\bar{b}\bullet\bar{a}\right)$ and Curl $\left\{\left(\bar{r}\times\bar{a}\right)\times\bar{b}\right\}=\bar{b}\times\bar{a}$ where $\bar{a}$ and $\bar{b}$ are constant vectors.

17) Evaluate $\int_C \bar{F}\bullet d\bar{r}$ where $\bar{F}=(x^2+y^2)\,\bar{i}-2xy\,\bar{j}$ and C is the rectangle in xy plane bounded by the lines x=0, x=a, y=0, y=b.

(OR)

18) Evaluate $\int_S \bar{F}.\bar{N}\,ds$ where $\bar{F}=18z\bar{i}-12\bar{j}+3y\bar{k}$ and S is the part of the plane 2x+3y+6z=12 located in the first octant.

19) State and prove Gauss divergence theorem.

(OR)

20) Verify Green's theorem for $\oint_C (xy+y^2)dx+x^2dy$ where C is the closed curve of the region bounded by $y=x$, $y=x^2$.